Final Project Presentation: Balancing Prism

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ME 4012 Modeling and Control of Motion Systems

Fall 2023 Final Project Presentation

Motivation

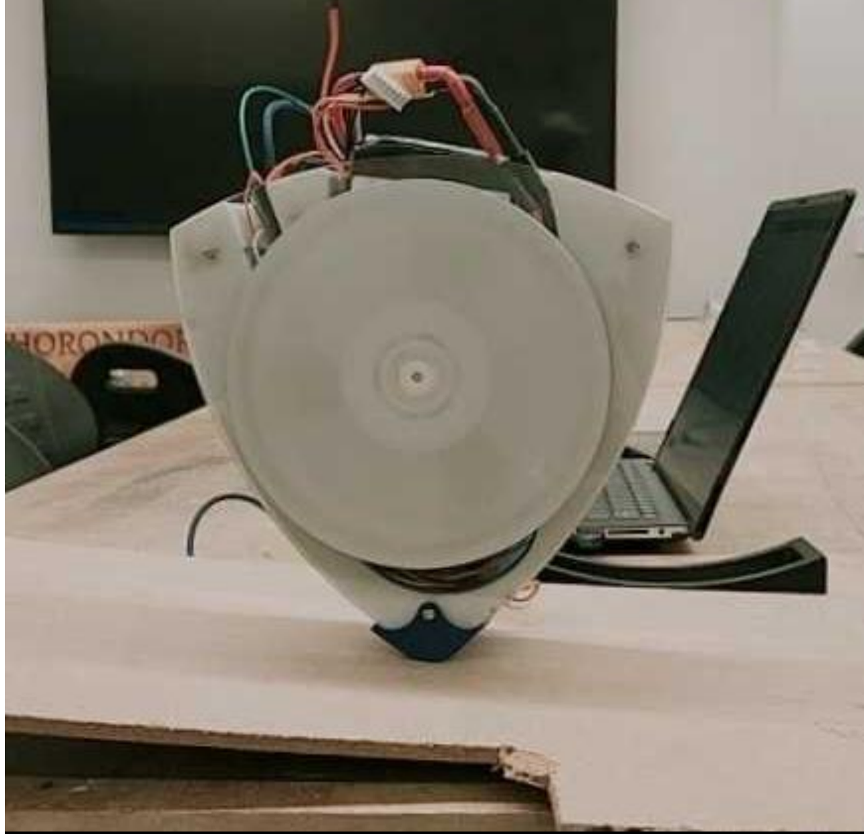
Objective: Balance prism on its unstable edge using reaction torque

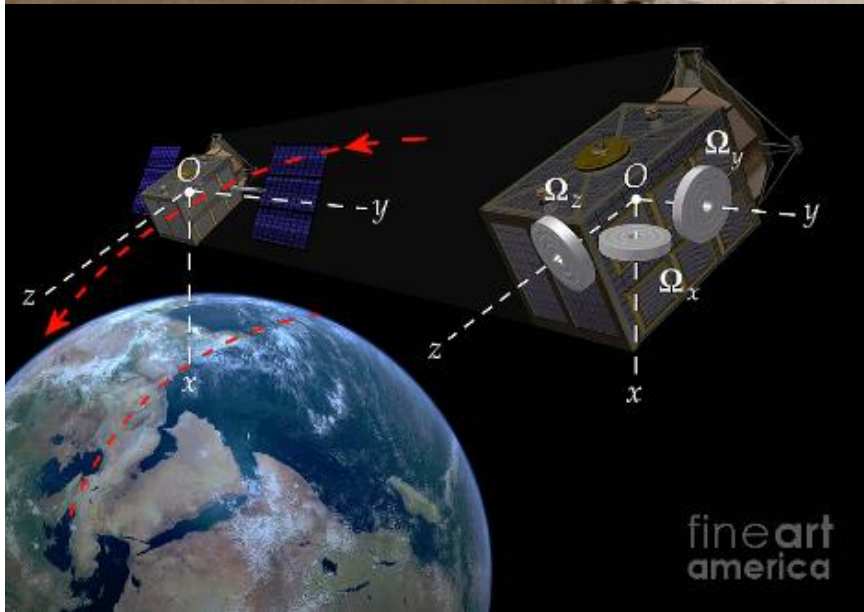
Applications:

- Satellite attitude control [1]
- Robot leg stabilization [2]
- Skyscraper gyroscope design [3]

Control body’s angular position using flywheel’s reaction torque

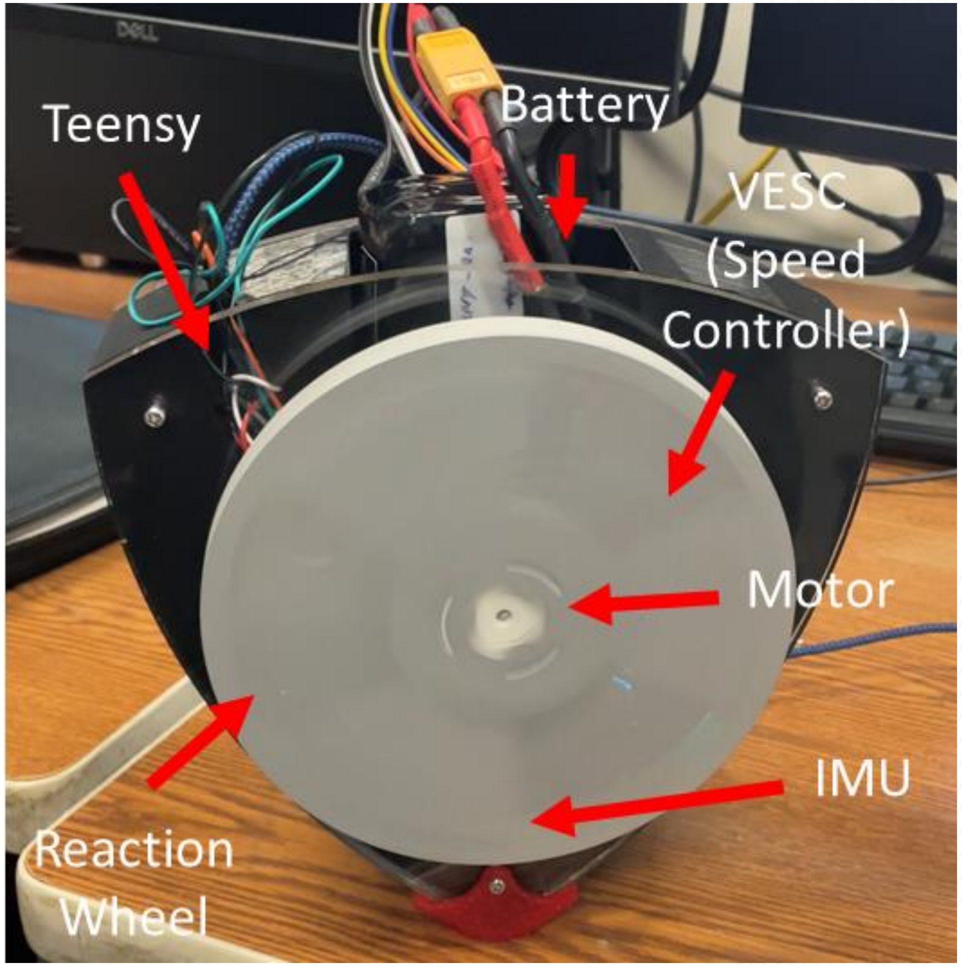
Can be applied to any inverted pendulum problem





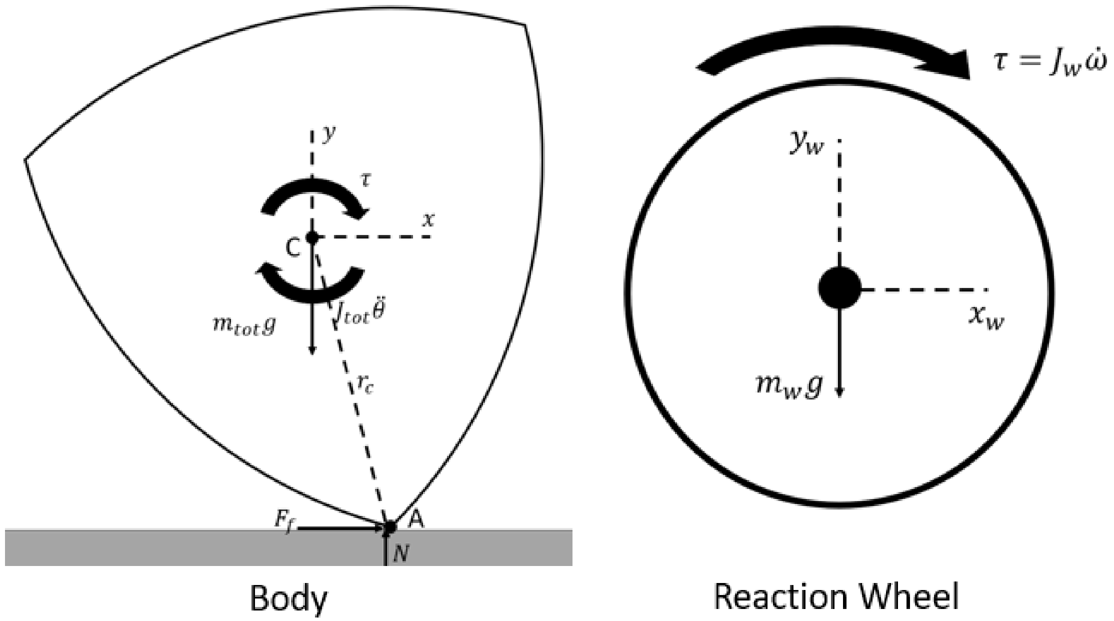
Design (Physical Assumptions)

- Overall shape is Reuleaux triangle
- Uses BLDC motor for increased torque and higher speed ceiling to avoid saturation
- Assumptions:
  - Center of mass of wheel and main body are approx. at the same location
  - Sufficient friction at contact point to assume no slip
- What Worked/Did Not Work:
  - Motor was incredibly powerful
    - Motor had dead zone in inputs
  - Original plastic flywheel lacked inertia
  - Original sharp edge was too unstable



Model

- Relates wheel velocity to body's acceleration using angular momentum approach
  - Computed about contact point A
- Used voltage-wheel velocity relationship to relate back to motor dynamics
  - Used simplified brushless DC motor model
    - Voltage-Velocity Constant



$$\frac{\Theta(s)}{V_{duty}(s)} = \frac{K_v(\frac{2\pi}{60})(J_w s)}{V_{max}(J_{tot}s^2 - m_{tot}gr_c)}$$

Plant

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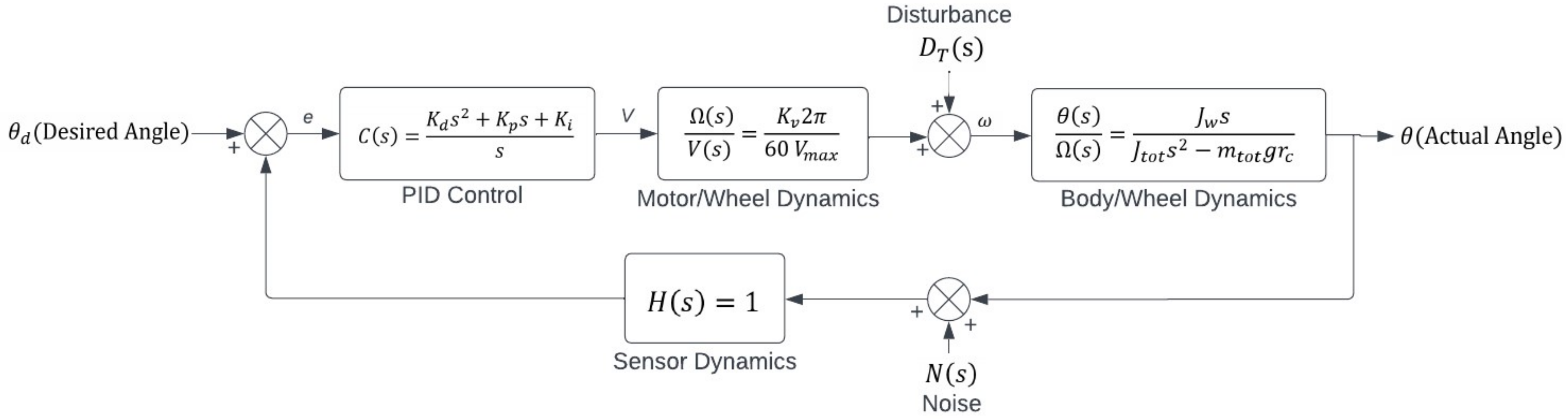
Model

Parameter	Kv	J_w	Voltage V_{max}	$J_{tot\,A}$	Mass m_{tot}	r_c
Value	1900	0.0017	25.2	0.0495	2.832	.117
Unit	RPM/V	Kg-m^2	V	Kg-m^2	Kg	m
Source	Spec Sheet	CAD	Spec Sheet	CAD	Scale	CAD

$$\frac{\Theta(s)}{V_{duty}(s)} = \frac{K_v(\frac{2\pi}{60})(J_w s)}{V_{max}(J_{tot}s^2 - m_{tot}gr_c)} = \frac{0.01323s}{0.04953s^2 - 3.25}$$

Controller Design – Block Diagram



Controller Design – Results

Desired characteristics:

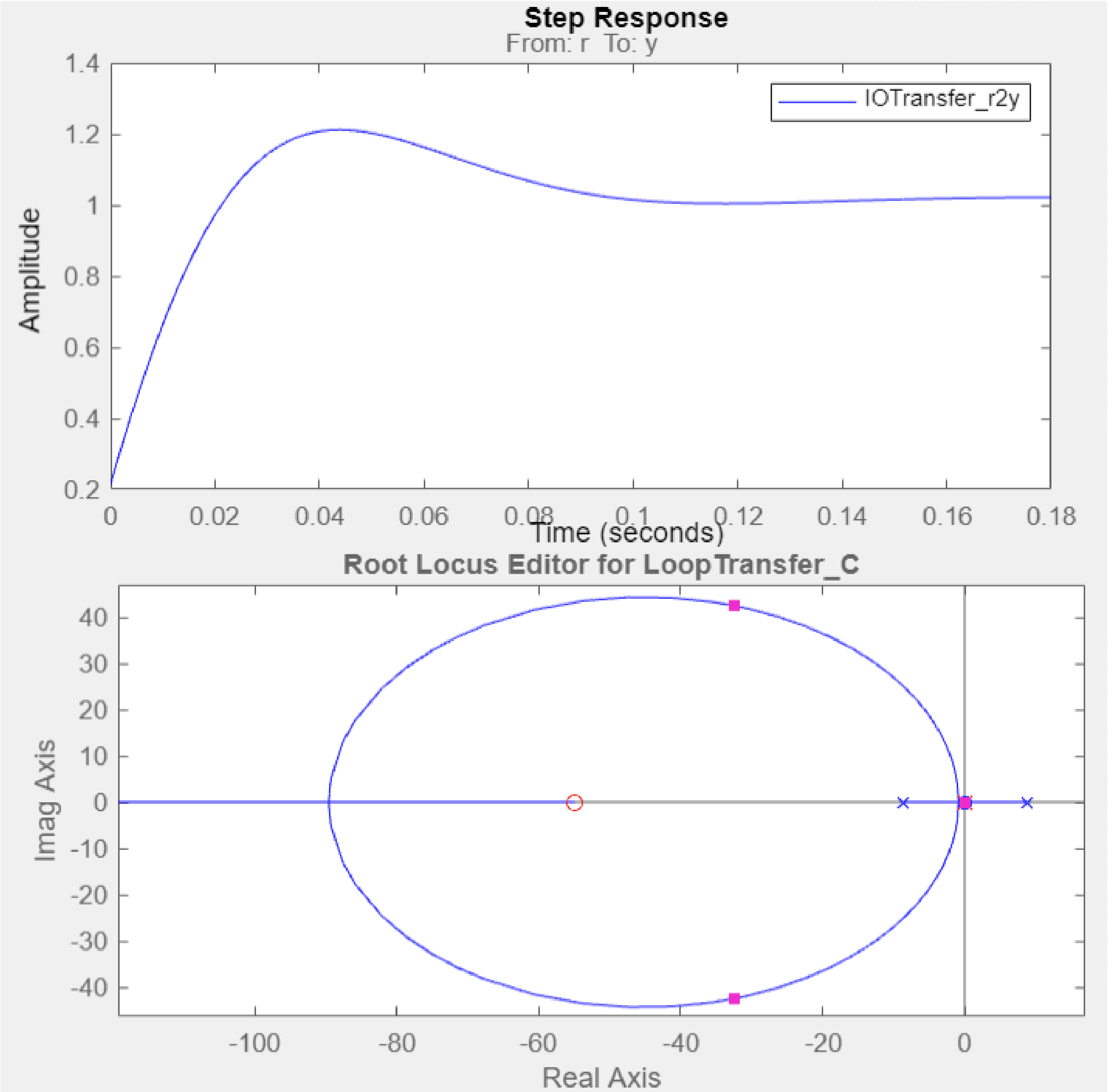
- <20% Overshoot
- Steady State Error < 0.75%
- Peak Time < 0.05 s

Design Techniques:

- Root Locus Pole Placement
  - Can be stabilized with PID

Theoretical gains:

- $K_p = 270.568$
- $K_i = 12{,}164$
- $K_d = 0.89435$



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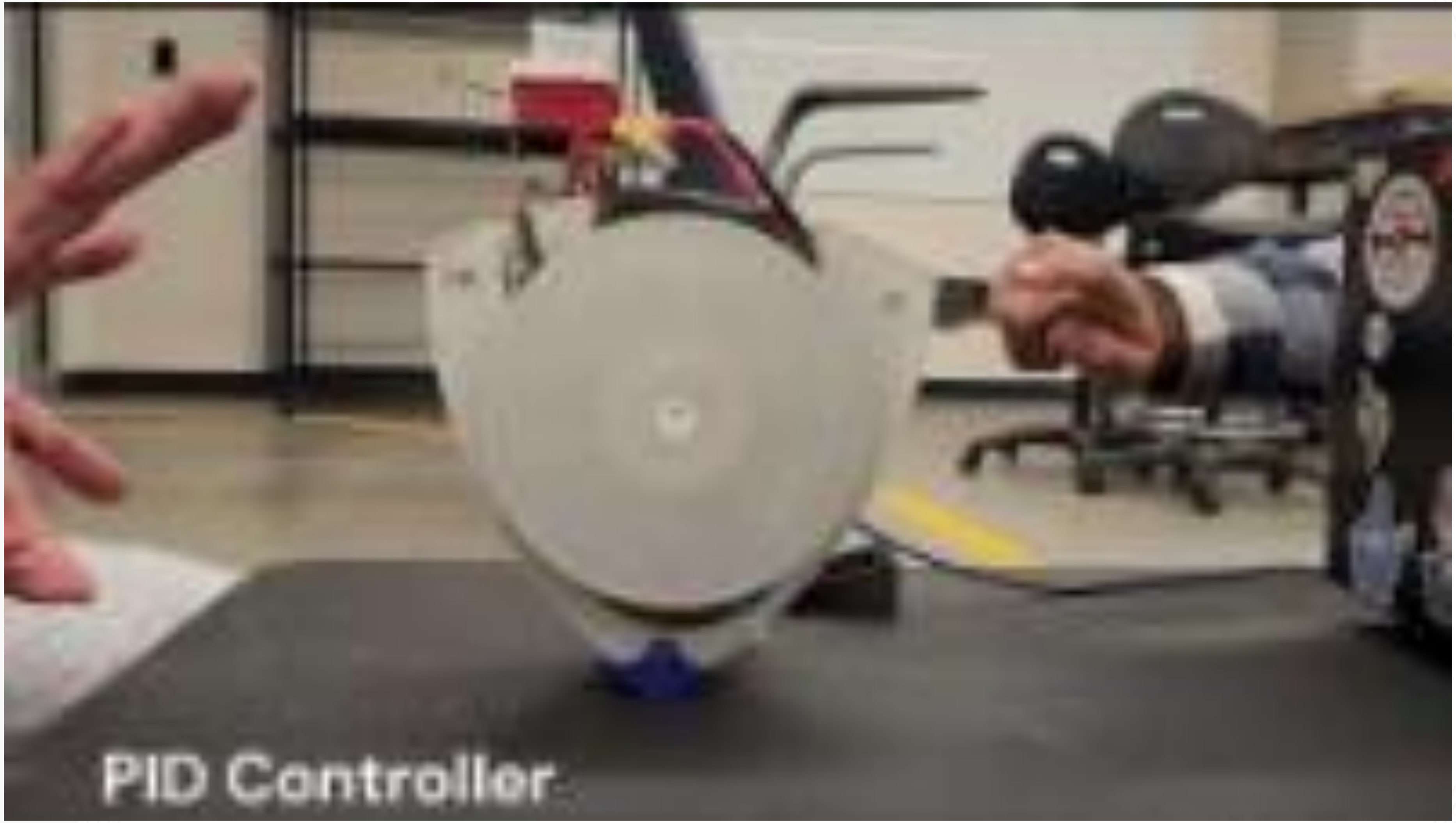
Video



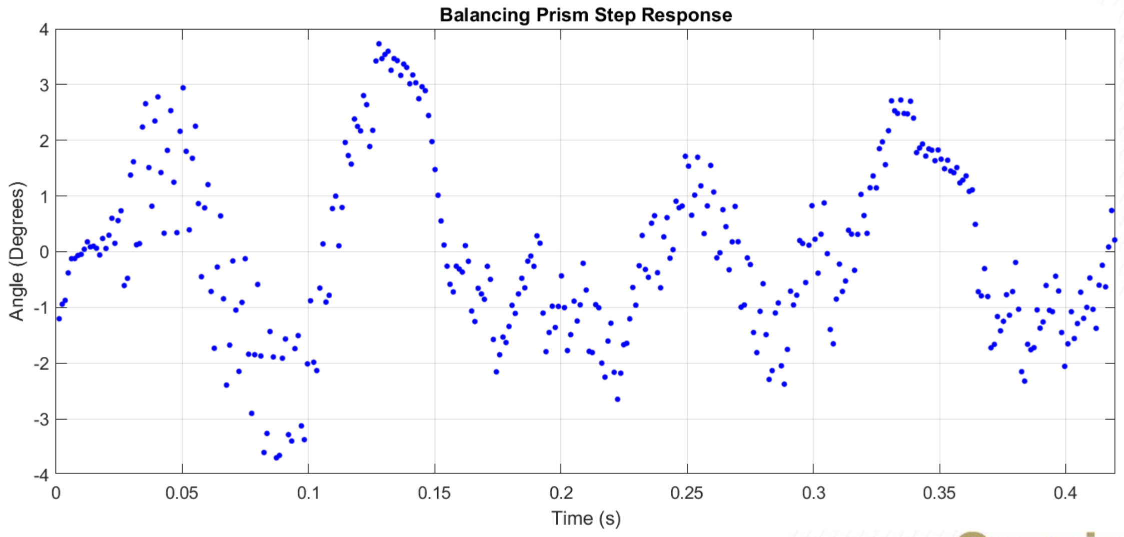
Experimental Results

- Code implemented using Arduino IDE to control using Teensy
  - Controller: PID
  - Actual Gains:
    - $K_p = 0.010$
    - $K_i = 0.072$
    - $K_d = 0.004$
- Angle reading was integrated from gyroscope readings

```
pComponent = Kp * (currentAngle - setPoint);
dComponent = angularVelocity * Kd;
iComponent = Ki * cumError;
dutyCycle = pComponent + dComponent + iComponent;
```

Control Law

- Varied greatly from theory
  - Caused by:
    - Motor dead zone
    - Nonlinear motor behavior
    - Sensor drift
    - Imperfect weight balance
    - Slight amounts of slip
  - Oscillatory behavior even when controlled



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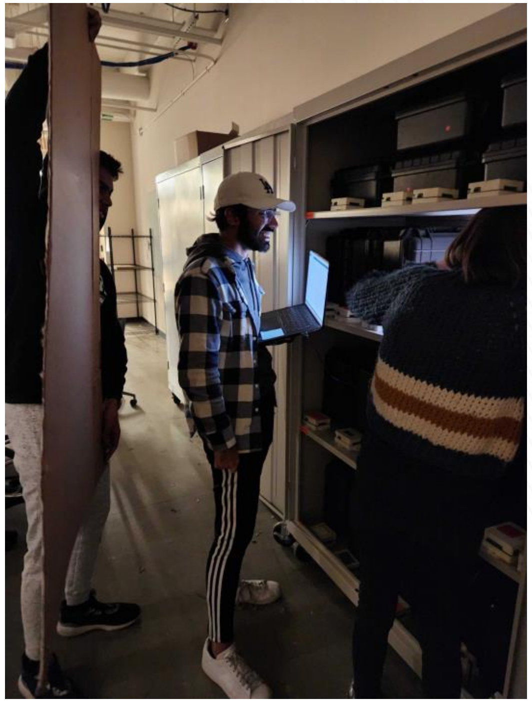
Conclusion

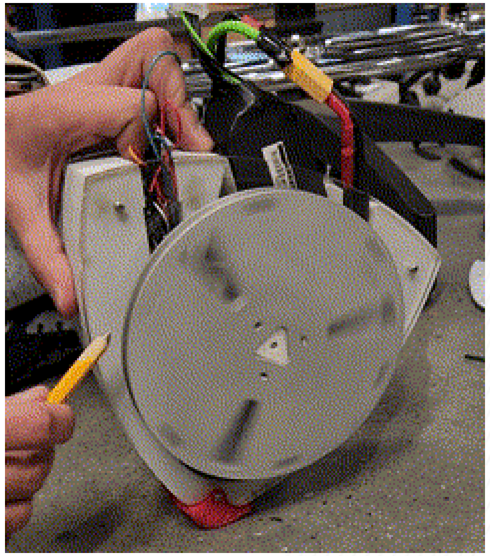
- Successfully balanced the triangle
  - Stays vertical despite ~30 degree tilt from horizontal
  - Exhibited disturbance rejection
  - PID Control
- Some discrepancies and nonlinearities between mathematical model and the actual model
  - Tuning controller corrected for issues
- Potential improvements
  - Brushed motor for more accurate modeling
  - Move flywheel above triangle COM

the easier way to balance a triangle

Special Thanks To

- Dr. Mazumdar and Noah for all their help
- Dr. Ferri for saying our first idea was horrifying to model
- The pencil that we used to reset our Teensy and sharpened
- The corpse of the combat robot we ripped the electronics from
- The terrified faces of the onlookers in the Invention Studio when the motor went to 50% power
- The 2110 trifold that Mannan used to shield the glow in the dark triangle from the light





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References

[1] Bitar, Andreea "Understanding Reaction Wheels" https://aerospace.honeywell.com/us/en/about-us/blogs/understanding-reaction-wheels#:~:text=Reaction%20wheels%20are%20an%20integral,torque%2C%20like%20rockets%20or%20propellants.

[2] C. Lee, “Enhanced Balance for Legged Robots Using Reaction Wheels,” *Robotic Exploration Lab*. https://roboticexplorationlab.org/papers/reaction_wheel.pdf

[3] H. Higashiyama, M. Yamada, Y. Kazao, and M. Namiki, “Characteristics of active vibration control system using gyro-stabilizer,” Engineering Structures, https://www.sciencedirect.com/science/article/abs/pii/S014102969700076X#:~:text=The%20gyro%2Dstabilizer%2C%20which%20has,low%20levels%20of%20wind%20excitations. (accessed Nov. 29, 2023).

Appendix A: Model Derivation

Variable Definitions

Θ: rotation angle of the full assembly measured from vertical

m_{tot}: total mass of the full assembly

r_c: distance from the center of mass C to the ground contact point A

J_w: moment of inertia of the wheel about C

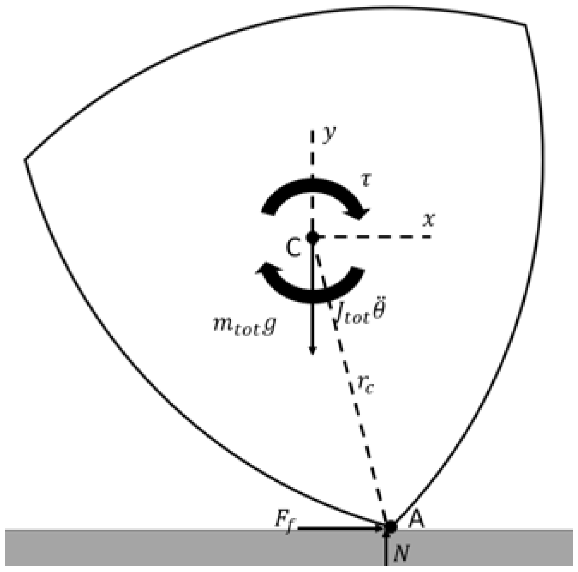
J_{tot}: moment of inertia of the full assembly about C

ω: angular velocity of the wheel in the body-fixed frame, defined as positive in the opposite direction from Θ

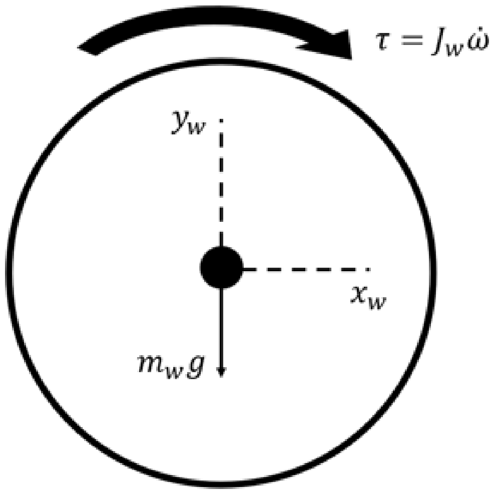
$J_{tot,A}$: moment of inertia of the fully assembly about A

K_v: velocity constant of the motor in RPM/V

V_{max}: voltage of battery; maximum voltage available



Body



Reaction Wheel

Appendix A: Model Derivation

Assumptions

1. There is sufficient friction to prevent slip at contact point *A*.
2. The centers of mass of the wheel and the main body are coincident with each other and with the motor shaft.
3. The dynamics can be linearized about $\theta = 0$ such that $\sin(\theta) \approx \theta$ and $\cos(\theta) \approx 1$.

Appendix A: Model Derivation

Model Derivation

The relationship between the sum of external moments on a multibody system and the time derivative of the total angular momentum of that system is defined by

$$\sum \overrightarrow{M_A} = \dot{\overrightarrow{H_C}} + \overrightarrow{r_{C/A}} \times m_{tot}\overrightarrow{a_C}. \qquad [1]$$

Since the reaction torque is an internal moment, the only external moment applied to the balancing prism is by gravity such that

$$\sum \overrightarrow{M_A} = m_{tot} g r_c \sin(\theta)\,\hat{k} \approx m_{tot} g r_c \theta \hat{k}. \qquad [2]$$

The righthand side of [1] becomes

$$\dot{\overrightarrow{H_C}} + \overrightarrow{r_{C/A}} \times m_{tot}\overrightarrow{a_C} = \left(-J_w\dot{\omega} + J_{tot}\ddot{\theta}\right)\hat{k} + r_c(-\sin(\theta)\,\hat{\imath} + \cos(\theta)\,\hat{\jmath}) \times m_{tot} r_c(-\ddot{\theta}\hat{\imath} + \dot{\theta}^2\hat{\jmath})$$

which linearizes and simplifies to

$$\dot{\overrightarrow{H_C}} + \overrightarrow{r_{C/A}} \times m_{tot}\overrightarrow{a_C} = \left(-J_w\omega + J_{tot,A}\dot{\theta}\right)\hat{k}. \qquad [3]$$

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Appendix A: Model Derivation

[2] and [3] combined define the equation of motion for the system,

$$J_{tot,A}\ddot{\theta} + m_{tot} g r_c \theta = J_w \dot{\omega}, \quad [4]$$

which can be converted using Laplace transform to the transfer function

$$\frac{\Theta(s)}{\Omega(s)} = \frac{J_w s}{J_{tot,A} s^2 - m_{tot} g r_c}. \quad [5]$$

The relationship between the motor input voltage of a brushless direct current motor is defined by

$$\omega = \frac{2\pi}{60} K_v V, \quad [6]$$

which can be converted using Laplace transform to the transfer function

$$\frac{\Omega(s)}{V(s)} = \frac{2\pi}{60} K_v. \quad [7]$$

Because duty cycle control is desired, [7] is divided by the maximum voltage of the power source (i.e., the battery voltage), such that

$$\frac{\Omega(s)}{V_{duty}(s)} = \frac{\frac{2\pi}{60} K_v}{V_{max}}. \quad [8]$$

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Appendix A: Model Derivation

The final transfer function derived from [5] and [8] is

$$\frac{\Theta(s)}{V_{duty}(s)} = \frac{K_v\left(\frac{2\pi}{60}\right)(J_w s)}{V_{max}(J_{tot}s^2 - m_{tot} g r_c)}. \qquad [9]$$

Which let us achieve results like this: